HEAT TRANSFER IN THE VORTEX REGION FOR A LATERAL FLOW PAST A CYLINDER

A. I. Leont'ev and B. A. Ryagin

Zhurnal Prikladnoi Mekhaniki i Tekhicheskoi Fiziki, Vol. 7, No. 6, pp. 111-114, 1966

An attempt is made to generalize the analytic methods of boundary-layer theory to the case of heat transfer in the flow separation zone.

It is assumed that a layer which can be described by boundary-layer theory is formed on the rear face of the cylinder exposed to the relatively stable reverse vortex flow.

It is also assumed that the velocity gradient along the normal to the surface, which is present in the vortex flow, has no appreciable effect on the development of this layer.

NOTATION

d is the cylinder diameter; x is the distance measured from the stagnation point along the rear surface of the cylinder; $X = x/d$ is the dimensionless coordinate; δ_T^{**} is the energy thickness; δ^{**} is the momentum thickness; ρ is the density; ν is the kinematic viscosity; W_{01} is the velocity of the undisturbed flow; W_0 is the velocity on the boundary of the wall layer; f_d is the dimensionless injection parameter; b_* is the critical value of the thermal permeability parameter; R is the Reynolds number; P is the Prandtl number; S is the Stanton number; R_z is the Reynolds number calculated from the corresponding flow velocities and characteristic linear dimensions; T_0 is the temperature on the surface of the boundary layer; T_w is the temperature of the wall; T' is the temperature of the injected gas; j_w is the current density of the injected gas.

$$R_d = \frac{W_0 d}{\nu}, \quad R_{01} = \frac{W_{01} d}{\nu},$$

$$R_T^{**} = \frac{W_0 \delta_T^{**}}{\nu}, \quad R^{**} = \frac{W_0 \delta^{**}}{\nu},$$

$$R_* = \frac{W_0 x}{\nu}, \quad j_w = \rho_w W_w, \quad f_d = \frac{j_w}{\rho W_{01}} \sqrt{R_{01}}.$$

Analytic methods for heat and mass transfer calculations which are based on the theory of the boundary layer are, in general, valid only for nonseparating liquid flows. In many practical cases the surface occupied by the vortex region extends over a considerable distance. There are, however, no analytic methods at present for calculating friction and heat transfer under these conditions.

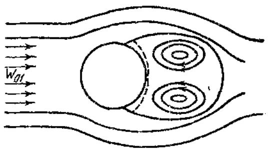

Fig. 1. Schematic illustration of flow past a cylinder.

It is well known that a vortex region with a relatively stable reverse motion is formed for sufficiently large Reynolds numbers when a cylinder is exposed to a lateral flow. It may be assumed as a first approximation that thermal and dynamic boundary layers described by the usual boundary-layer theory appear on the surface of the cylinder as a result of the flow (Fig. 1). It may also be supposed that the velocity gradient which exists in a vortex flow of this kind along the normal to the wall has no appreciable effect on the development of the boundary layer. The velocity on the outer surface of the boundary layer can be determined by the method put forward in [5]. The results of analytic calculations of the transverse flow past a cylinder are described quite well by the interpolation formula

$$W_0 = A W_{01} \operatorname{th} \alpha X, \tag{1}$$

where W_0 is the velocity on the outer surface of the boundary layer at the rear of the cylinder.

For a circular cylinder $A = 2.93$ and $\alpha = 1.12$. The equation for the thermal boundary layer will be written in the form

$$\frac{dR_T^{**}}{dX} + \frac{R_T^{**}}{\Delta T}\frac{d\Delta T}{dX} = \frac{1}{2} R_d S_0. \tag{2}$$

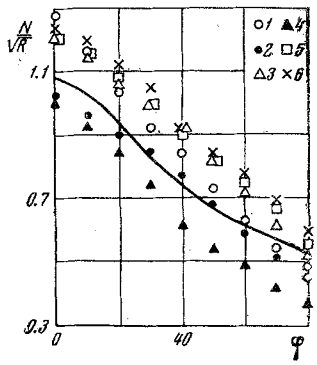

Fig. 2. Comparison of calculations based on Eq. (3) with experimental data (impermeable cylinder). Kruzhilin's experiment [1]: 1) $R = 3.51 \cdot 10^4$, 2) $R = 1.4 \cdot 10^4$; experiments of Schmidt and Wenner [2]: 3) $R = 1.01 \cdot 10^5$, 4) $R = 3.98 \cdot 10^4$; experiments of Johnson and Hartnett [3]: 5) $R = 1.01 \cdot 10^5$; experiments of Eckert et al. [4]: $R = 1.3 \cdot 10^5$.

If we take the heat transfer law in the form

$$S_0 = \frac{0.22}{P^{4/3} R_T^{**}},$$

and take Eq. (1) into account for the case $q_w = \text{const}$, we have

$$R_T^{**} = \sqrt{0.11}\, R_{0.1}^{0.5} P^{-4/3} \sqrt{X}.$$

Hence,

$$N = \sqrt{0.44 A\alpha}\, R_{01}^{0.5} P^{1/3} \left(\frac{\operatorname{th} \alpha X}{\alpha X}\right)^{1/2}. \tag{3}$$

Correspondingly, for T = const we have

$$N = \sqrt{0.22 A\alpha}\, R_{01}^{0.5} P^{1/3} \frac{\operatorname{th} \alpha X}{\sqrt{\ln \operatorname{ch} \alpha X}}.$$

For the more general case in which $\Delta T = \Delta T_0(1 + kX)^n$ $(\Delta T = T_0 - T_w)$ we have

$$R_T^{**} = \sqrt{\frac{0.22A}{\alpha}\frac{R_{01}^{0.5} P^{-4/3}}{(1+kx)^n}} \times$$

$$\times \Big\{ \ln \operatorname{ch} \alpha X + C_{2n}^{1} \frac{k}{\alpha} \sum_{i=1}^{\infty} \frac{2^{2i}(2^{2i}-1) B_{2i}}{(2i+1)(2i)!} (\alpha X)^{2i+1} + \ldots$$

$$\ldots + C_{2n}^{2n} \left(\frac{k}{\alpha}\right)^{2n} \sum_{i=1}^{\infty} \frac{2^{2i}(2^{2i}-1) B_{2i}}{(2i+2n)(2i)!} (\alpha X)^{2i+2n} \Big\}^{1/2},$$

$$N = \sqrt{0.22 A\alpha}\, R_{01}^{0.5} P^{1/3} (1+kX)^n \operatorname{th} \alpha X \times$$

$$\times \Big\{ \ln \operatorname{ch} \alpha X + \ldots + C_{2n}^{2n} (k/\alpha)^{2n} \times$$

$$\times \sum_{i=1}^{\infty} \frac{2^{2i}(2^{2i}-1)B_{2i}}{(2i+2n)(2i)!}(\alpha X)^{2i+2n}\Big\}^{-1/2},$$

where Ξ is the convergent series

$$\Xi = \sum_{i=1}^{\infty} \frac{2^{2i}(2^{2i}-1)B_{2i}}{(2i+p)(2i)!}(dX)^{2i+2n} \qquad (|X|<\pi,\ p>-1).$$

It may be expected that the laminar boundary layer will go over into a turbulent state at some point along the surface. According to [6], this transition occurs for $R^{**} = 240$. The value of X corresponding to the transition from the laminar boundary layer to the turbulent layer can readily be calculated with the aid of the conditions $dp/dx = 0$ and $p \approx 1$. When ΔT = const we have $X_n = 1.09$, and when q_w = const we have $X_n = 0.92$.

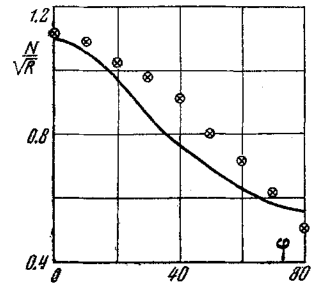

Fig. 3. Comparison of Eq. (8) with the experiments of Eckert et al. (helium gas), $f_d = 0.08$.

We shall take the heat transfer in the turbulent boundary layer in the form

$$S = {}^1/_2\, BP^{-0.75} R_T^{**-m}, \qquad (4)$$

so that when $T = \Delta T_0[1 - k(X - X_0)]^n$ we have from Eqs. (2) and (4)

$$(R_T^{**})^{1+m} = \frac{1}{[1-k(x-x_0)]^n} \times$$

$$\times \left\{(R_{T_0}^{**})^{1+m} + \frac{R_{01}{}^1/_4 BP^{-0.75}(1+m)A}{[n(1+m)+1]k} \times\right.$$

$$\left.\times [1-\{1-k(x-x_0)\}^{n(1+m)+1}]\right\},$$

$$N = P^{1/4} R_{01} A \frac{B}{2}[1-k(x-x_0)]^{nm}\left\{R_{T_0}^{**(1+m)} + \right.$$

$$+ \frac{R_{01}{}^1/_4 BP^{-0.75}(1+m)A}{[n(1+m)+1]k} \times$$

$$\left.\times [1-\{1-k(x-x_0)\}^{n(1+m)+1}]\right\}^{-\frac{m}{1+m}}.$$

An analogous analysis can be performed for the lateral flow past a permeable cylinder.

The equation for the thermal boundary layer under these conditions is of the form [5]

$$\frac{dR_T^{**}}{dX} + \frac{R_T^{**}}{\Delta T}\frac{d(\Delta T)}{dX} =$$

$$= \frac{1}{2} R_d S_0(\Psi_s + b) \qquad \left(\Psi_s = \frac{S}{S_0},\ b = \frac{j_w}{\rho W_0}\frac{1}{S_0}\right), \qquad (5)$$

where Ψ_s is the relative Stanton number, and b is the thermal permeability parameter. From the heat balance we have

$$\Psi_s = k_1 b, \qquad \left(k_1 = \frac{T - T_w}{T_0 - T_w}\right).$$

When j_w = const, T_w = const, and T_0 = const, we have from Eq. (5)

$$R_T^{**} = \frac{1}{2} R_d a_0 (k_1 + 1) X \qquad \left(a_0 = \frac{j_w}{\rho W_0}\right).$$

Since $S = \Psi_s S_0$, and $S_0 = 0.22\, R_T^{**-1} P^{-4/3}$, it follows that

$$S = \sqrt{0.22}\, R_*^{-0.5} P^{-2/3} \frac{\Psi_s}{(\Psi_s + b)^{1/2}}$$

Since $(0.22)^{1/2} P^{-2/3} R_*^{-0.5} = S_0$ for q_w = const, we have

$$\Psi_{R_*} = \left(\frac{S}{S_0}\right)_{R_*} = \frac{\Psi_s}{(\Psi_s + b)^{1/2}}$$

On the other hand, $\Psi_{R_*} = k^1 b'$, in which case

$$k_1 b_T' = \frac{k_1 b}{(\Psi_s + b)^{1/2}} \left(\Psi_{R_*} = \left(\frac{S}{S_0}\right)_{R_*},\ b' = \frac{j_w}{\rho W_0}\frac{1}{S_{0*}}\right), \qquad (6)$$

where Ψ_{R_*} is the ratio of the Stanton numbers for R_* = idem.

According to Eckert et al. [7], for a laminar boundary layer we have

$$\Psi_{R_*} = 1 - b'/b_k'. \qquad (7)$$

Using Eqs. (6) and (7) it can be shown that

$$N = N_{0q_w}\left[1 + \frac{P^{2/3} f_d}{\sqrt{0.44A\alpha}}\left(\frac{\alpha X}{\operatorname{th}\alpha X}\right)^{1/2}\left(1 - \frac{1}{b_k'}\right)\right]^{-1}$$

$$(q_w = \text{const}). \qquad (8)$$

Figures 2–4 give a comparison between the above analytic formulas and experimental data. Figure 2 shows the distribution of the local values of the Nusselt number at the rear of a cylinder with an impermeable surface. The solid curve corresponds to calculations based on Eq. (3). It is clear from the figure that the experimental points reported by different authors tend to lie on the analytic curve in practically the entire rear region. Similar results are obtained for a cylinder with a permeable surface in a flow of helium (Fig. 3). Less satisfactory results are obtained when Eq. (8) is compared with the results reported in [3] for air (Fig. 4). The discrepancy increases with increasing amount of injected gas. It is possible that in this case the effect of the injected gas on the velocity distribution in the vortex flow must be taken into account. The above analysis shows that

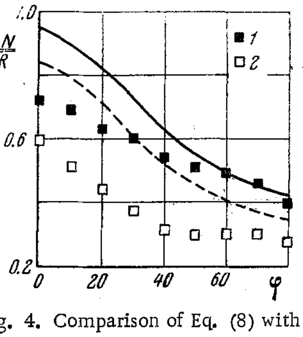

Fig. 4. Comparison of Eq. (8) with the experiments of Johnson and Hartnett (air): solid curve and filled-in squares correspond to $f_d = 0.808$; dashed curve and open squares correspond to $f_d = 1.808$.

methods of the boundary-layer theory can be used with adequate success for calculating the heat transfer in the vortex region.

REFERENCES

1. G. N. Kruzhilin, "Heat transfer in the case of a circular cylinder in a transverse air flow," Zh. tekhn. fiz., vol. 8, no. 2, 1938.

2. H. Schlichting, Boundary Layer Theory [Russian translation], Izd. inostr. lit., 1956.

3. D. V. Johnson and J. P. Hartnett, "Heat transfer from a cylinder in crossflow," J. Heat Transfer, ASME, no. 2, 1963.

4. O. E. T'yufik, E. R. G. Eckert, and L. S. Yurevich, "Effect of thermal diffusion on heat transfer in lateral flow past the cylinder," Raketnaya tekhnika i kosmonavtika, vol. 1, no. 7, 1963.

5. Collection: Heat Transfer and Friction in a Turbulent Boundary Layer [in Russian], Izd. SO AN SSSR, 1964.

6. C. C. Lin, Theory of Hydrodynamic Stability [Russian translation], Izd. inostr. lit., 1958.

7. E. R. G. Eckert, A. A. Kheidei, and V. K. Linkovich, "Heat transfer, recovery temperature and skin friction on a flat plate with hydrogen supplied to the laminar boundary layer," collection: Heat and Mass Transfer (ed. A. V. Luikov and B. M. Smol'skii) [in Russian], Gosenergoizdat, 1963.

14 October 1965 Novosibirsk